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Closed strings tachyons and non-commutative instabilities

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Abstract: We observe a relation between closed strings tachyons and one-loop instabilities in non-supersymmetric non-commutative gauge theories. In particular we analyze the spectra of type-IIB string theory on $\mathbb{C}^3/\mathbb{Z}_N$ orbifold singularities and the non-commutative field theory that lives on D3 branes located at the singularity. We find a surprising correspondence between the existence or not of one-loop low-momentum instabilities in the non-commutative field theory and the existence or not of tachyons in the closed string twisted sectors. Moreover, the relevant piece of the non-commutative field theory effective action is suggestive of an exchange of closed string modes. This suggests that non-commutative field theories retain some information about the dynamics of the underlying string configuration. Finally, we also comment on a possible relation between closed string tachyon condensation and field theory tachyon condensation.

Keywords: Superstrings and Heterotic Strings, D-branes, Tachyon Condensation.

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Contents

1. Introduction

The issue of closed string tachyon condensation is one of the most difficult problems in string theory. Bosonic string theory contains a tachyon, but it does not necessarily mean that the theory is inconsistent, although the ultimate fate of its decay process is still mysterious.

Recently there has been a lot of progress in the understanding of tachyon condensation processes in non-supersymmetric string theories with worldsheet supersymmetry [1, 2]. The prime example is IIB string theory on geometric orbifold singularities that break spacetime supersymmetry. In this case the twisted sector tachyons live on the singularity. In several cases, it was argued that the twisted sector tachyons condense and the true vacuum of the theory is type IIB on flat background [1]. Another example is type 0 string theory, which can be thought of as an orbifold of type II. The tachyon in this case is ten-dimensional. Although the meaning of this instability is not completely clear, there are arguments in favor of this tachyon condensation process having type II theory as the corresponding endpoint [2].

Another, seemingly unrelated subject, is the study of non-commutative gauge field theories (see [3, 4] for recent reviews on the subject). These theories exhibit many similarities to open string theory: they contain dipoles as fundamental objects, there is a connection between the UV and the IR (UV/IR mixing) [5, 6] and the gauge group and its representation are constrained [7]–[10]. In addition, due to lack of Lorentz invariance the gauge field often becomes tachyonic (or massive), since both gauge invariance and Lorentz symmetry are needed in order to protect the masslessness of the gauge boson [11]–[13]. More precisely, one-loop effects modify the dispersion relation of gauge bosons in the center of the group. The modes polarized along the non-commutative directions verify

$$E^2 = \vec{p}^{\,2} - \lambda \frac{N_b - N_f}{\pi^2} \frac{1}{\tilde{p}^2} \,, \tag{1.1}$$

where λ is the 't Hooft coupling; $\tilde{p}^\mu = \theta^{\mu\nu} p_\nu$ and $\theta^{\mu\nu}$ is a matrix that parametrizes the noncommutativity of the space $[x^\mu, x^\nu]_* = i\theta^{\mu\nu}$. To avoid problems with unitarity [14], we

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will always consider $\theta^{\mu\nu}$ with only space indices, i.e. $\theta^{0i} = 0$. In the previous equation N_b and N_f are the number of bosonic and fermionic degrees of freedom in the adjoint representation. In particular, when there are more bosons than fermions, the low momentum modes of the gauge bosons in the center of the group become unstable.

In this work we consider non-commutative field theories arising on the world-volume of D-branes in the decoupling limit, and observe a relation between the non-commutative field theory instabilities and the closed string tachyons in the parent string theory. In a previous work [15] it was found that the piece of the one-loop effective action containing the instability, for the non-commutative field theory arising on a collection of D3 branes of type 0 string theory, has a form suggestive of an origin from exchange of the closed string modes. There were also some hints that the instability associated to the gauge boson in the field theory is related to the presence of the bulk tachyon. However, the relation between the two could not be made precise.

In the present work we generalize this observation to an infinite class of models, and suggest that the two (seemingly very different!) kinds of instabilities are related. We consider type-IIB string theory on geometric orbifold singularities, with a constant background of NS-NS 2-form field in the untwisted directions. We analyze two quantities: the spectrum of the lowest modes of the closed string tower in the different twisted sectors, and the one-loop correction to the dispersion relation of the gauge bosons in the non-commutative field theory that lives on a collection of M D3 branes sitting at the orbifold singularity. The closed string contains N sectors: untwisted sector and $N-1$ twisted sectors. The various low-lying modes (in the NS-NS sector) in each sector are either massless, massive or tachyonic. The generic non-commutative field theory is a $\mathrm{U}(n)^N$ gauge theory. We can take linear combinations of the N center gauge bosons which naturally couple to the different closed string twisted sectors of the underlying string theory picture. These linear combinations are not mixed by the one-loop mass correction (they diagonalize the corresponding mixing matrix) and can remain massless at one-loop, or become massive or tachyonic — according to (1.1).

We find that there is a one-to-one correspondence between the two systems, in the following sense: The non-commutative field theory produces one-loop instabilities precisely for those linear combinations coupling to closed string twisted sectors containing tachyons; the one-loop correction exists but does not lead to instabilities for linear combinations coupling to closed string twisted sectors which are non-supersymmetric but do not contain tachyons; finally, the one-loop correction is absent precisely for linear combinations coupling to closed string twisted sectors which are supersymmetric. Thus, the one-loop non-commutative dynamics that leads to this rich pattern of field theory mass corrections seems to correlate with properties of the closed sector of the underlying string theory. In other words, the non-commutative field theory seems to retain some crucial aspects of the closed string dynamics!

We moreover show that the full form of the one-loop effective action of the non-commutative theory is very reminiscent of the exchange of closed strings. This and the previous observation might lead to the speculation that tachyon condensation processes in the two systems are also related. Unfortunately present techniques do not allow to explore the instability of non-commutative field theory to the point of really testing this beautiful, but far stronger, form of a correspondence

The organization of the paper is as follows: in section 2 we present our main result. We analyze the (one-loop) field theory spectrum and we compare it to the (tree-level) lowest modes of the closed string spectrum. We show the correlation between the existence of non-commutative field theory instabilities and closed string twisted tachyons. In section 3 we show that the one-loop effective action for the field theory that lives on orbifold singularities is reminiscent of an exchange of twisted sector modes. Finally, in section 4 we discuss our results and comment about the possibility of tachyon condensation in the field theory and the relation to closed strings dynamics.

2. Closed strings spectrum versus field theory spectrum

In this section we will compare the closed string spectrum of type-IIB string theory on orbifold singularities to the field theory spectrum on D3-branes sitting at the tip of the singularity. We will calculate first the one-loop correction to the mass of the gauge bosons in the field theory side and then we will compare it to the masses of the lowest modes in the closed string tower.

We consider type-IIB theory on a generic $\mathbb{C}^3/\mathbb{Z}_N$ orbifold (although we expect our results to extend straightforwardly to other orbifold groups, including non-abelian ones). Regarding 10d flat space as $M_4 \times \mathbb{R}^6$, the Lorentz group decomposes as $\mathrm{SO}(1,9) \to \mathrm{SO}(1,3) \times \mathrm{SO}(6)$. The orbifold action is a discrete subgroup of this SO(6). Its action can be specified by defining how it acts on the spinor representation 4 of $\mathrm{SO}(6) \sim \mathrm{SU}(4)$. This is given by an order N matrix in SU(4), which can be taken diagonal

$$\mathrm{diag}\left(e^{2\pi i a_1/N}, e^{2\pi i a_2/N}, e^{2\pi i a_3/N}, e^{2\pi i a_4/N}\right), \qquad \sum_\alpha a_\alpha = 0 \bmod N\,. \tag{2.1}$$

The integers a_α are defined mod N. Coordinates in R^6 transform in the vector representation 6 of SO(6), which is obtained from the antisymmetric tensor product of two 4's. Taking complex coordinates, the geometric action of Z_N on the coordinates of $\mathbb{C}^3$ is

$$Z^\beta \to e^{2\pi i b_\beta/N} Z^\beta \qquad \beta = 1, 2, 3\,, \tag{2.2}$$

where the b_β are determined by the a_α as

$$b_1 = a_2 + a_3\,, \qquad b_2 = a_3 + a_1\,, \qquad b_3 = a_1 + a_2\,. \tag{2.3}$$

$\mathbb{C}/\mathbb{Z}_N$ and $\mathbb{C}^2/\mathbb{Z}_N$ orbifold models are obtained when two or one b_β's respectively are zero. The case $b_\beta = 0(\mathrm{mod} N)$ but a_α non-trivial corresponds to type 0 string theory. Type 0 orbifolds are also included among our models.

We place now n D3-branes[1] at the fixed point of the orbifold. The tree-level spectrum of the field theory on the branes is obtained from the massless states of the open string sector. Using standard projection rules for the Chan-Paton wavefunctions [26], the field theory

[1] Here we refer to dynamical D3-branes, namely in the covering space, we consider Nn D3-branes in n copies of the regular representation.

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contains the following fields. The gauge group is $\mathrm{U}(n)^N$. The matter content consists of Weyl fermions transforming in the representation $(\Box_i, \overline{\Box}_{i+a_\alpha})$, for $\alpha = 1, \ldots, 4$, and complex scalars in $(\Box_i, \overline{\Box}_{i+b_\beta})$, for $\beta = 1, 2, 3$. The theory is generically non-supersymmetric. Interactions are easily determined by keeping the Z_N-invariant terms in the action of the parent $\mathcal{N} = 4$ supersymmetric theory of D3-branes in flat space. We will not need the explicit expressions here. However, it is important to realize that the gauge coupling constants of the different factors in the orbifold theory are all equal, since they are inherited from the unique gauge coupling in the parent theory of D3-branes in flat space. The same conclusion is reached by realizing that gauge couplings in the quotient theory are controlled by vevs of closed string twisted states, which are set to be equal at the point in moduli space which is described by a CFT orbifold. Being equal for all gauge factors, the gauge coupling g will factor out of all our gauge field theory computations below.

Turning on a 2-form background in two (spatial) directions of the M_4 makes the corresponding field theory non-commutative. The models we consider here are generically chiral. In the commutative case the U(1)'s are anomalous and therefore become massive via a Green-Schwarz mechanism and therefore decouple, leading to an $\mathrm{SU}(n)^N$ theory at low-energies [16]. In the present non-commutative case the situation is more subtle [17]: the U(1)'s become massive and decouple only for zero non-commutative momentum $\tilde{p} = 0$. For non-zero $\tilde{p}$ the anomaly vanishes. In the following we will consider only the latter case.

One of the most striking characteristics of non-commutative theories is the appearance of infrared divergences whose origin is the integration of high momenta in non-planar loops [5]. We would like to study the effects of pole-like infrared divergences associated to a non-commutative theory with the previous gauge group and matter content. These poles generate a contribution to the polarization tensor of the $\mathrm{U}(1)_i$ photons of the form $\Pi_{ij}^{\mu\nu} = M_{ij}(g^2 n/\pi^2)(\tilde{p}^\mu \tilde{p}^\nu/\tilde{p}^4)$, with $i, j = 1, \ldots, N$. The matrix M_{ij} can be read directly from the matter content as follows. Only matter transforming in the adjoint representation contributes to M_{ii}. In particular each bosonic degree of freedom contributes $+1$ and each fermionic degrees of freedom -1. When there is only one gauge group factor, for example if only one type of fractional brane is placed at the orbifold point, this reproduces the 1-loop corrected dispersion relation (1.1). M_{ij} with $i \neq j$ is determined by the bifundamental matter content. The rule is as before: each bosonic (fermionic) degree of freedom contributes $+1$ (-1). To our knowledge, the fact that bifundamental matter can give rise to non-planar diagrams has not been mentioned in the existing literature. However it is easy to see that this is the case if one uses 't Hooft's double line notation. It is possible to draw a non-planar graph for the vacuum polarization with bi-fundamental matter inside the loop, if the external legs belong to different U(1) factors. The contribution to the 1-loop effective action that summarizes these effects is

$$\Delta S = \frac{g^2}{2\pi^2} \int \frac{d^4 p}{(2\pi)^4} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \sum_{i,j=1}^{N} M_{ij} \,\mathrm{Tr}\, A_\mu^{(i)}(p) \mathrm{Tr}\, A_\nu^{(j)}(-p)\,, \tag{2.4}$$

where we have chosen the normalization for Lie algebra generators $\mathrm{Tr} t_a t_b = \delta_{ab}$, with

$t_0 = (1/\sqrt{n})\mathbf{1}$. The matrix M_{ij}, as explained, is given by

$$M_{ij} = 2\delta_{ij} - \sum_\alpha (\delta_{i,j-a_\alpha} + \delta_{i,j+a_\alpha}) + \sum_\beta (\delta_{i,j-b_\beta} + \delta_{i,j+b_\beta}) \,. \tag{2.5}$$

Note that in the above expression the vectors contribute twice as much as scalars, since they have two physical polarizations in four dimensions.

In order to analyze the potential instabilities arising from the pole-like contributions to the polarization tensor, we need to diagonalize the matrix M. This is achieved by the set of vectors $e^{(k)}$, $k = 0, \ldots, N-1$, with components $e_j^{(k)} = e^{2\pi i\, jk/N}$, $j = 1, \ldots, N$. The associated eigenvalues are

$$\epsilon^{(k)} = 2\left(1 - \sum_\alpha \cos\frac{2\pi k a_\alpha}{N} + \sum_\beta \cos\frac{2\pi k b_\beta}{N}\right) = 16 \prod_\alpha \sin\frac{\pi k a_\alpha}{N} \,, \tag{2.6}$$

where, using the freedom to redefine the a_α modulo N, we have set $a_4 = -(a_1 + a_2 + a_3)$. Thus the effective action can be rewritten as

$$\Delta S = \frac{g^2}{2\pi^2} \int \frac{d^4 p}{(2\pi)^4} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \sum_{k=0}^{N-1} \epsilon^{(k)}\, B_\mu^{(N-k)}(p)\, B_\nu^{(k)}(-p) \,, \tag{2.7}$$

with

$$B_\mu^{(k)} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{2\pi i\, jk/N}\, \mathrm{Tr}\, A_\mu^{(j)} \,. \tag{2.8}$$

Notice that $B_\mu^{(k)\dagger} = B_\mu^{(N-k)}$ and, consequently, $\epsilon^{(k)} = \epsilon^{(N-k)}$. The tree level kinetic term for the center gauge bosons remains diagonal in terms of the $B_\mu^{(k)}$'s. Thus we may write a one-loop corrected dispersion relation for the modes of $B_\mu^{(k)}$ polarized along the non-commutative directions, which read

$$E^2 = \vec{p}^2 - g^2 n \frac{\epsilon^{(k)}}{\pi^2} \frac{1}{\tilde{p}^2} \,. \tag{2.9}$$

Adjoint scalars, in case they are present, also get pole-like infrared contributions to their self-energy. The couplings of the adjoint scalars to the other fields can be obtained by dimensional reduction from a theory in $D > 4$ dimensions and where the only adjoint bosons are the gauge fields. Using this, we obtain the contribution to the effective action (see [15] for a detailed derivation for the case of type 0 D3-branes)

$$\Delta S' = \frac{g^2}{4\pi^2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{\tilde{p}^2} \sum_{i,j=1}^{N} M_{ij} \sum_{l=1}^{D-4} \mathrm{Tr}\, \phi_l^{(i)}(p) \mathrm{Tr}\, \phi_l^{(j)}(-p) \,, \tag{2.10}$$

where ϕ_l denote the adjoint (real) scalars. The same diagonalization procedure applied to ΔS can by used for $\Delta S'$. Only adjoint bosons (vectors or scalars) get corrected at one loop. Bi-fundamental bosons are not corrected. The reason, clearly seen in 't Hooft double line notation, is that one cannot draw a non-planar graph for the vacuum polarization if

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the external legs are in the bi-fundamental representation. Note also that fermions (in any representation) do not get corrections.

We will now show that there is a remarkable relation between the sign of $\epsilon^{(k)}$ and the sign of the $(\text{mass})^2$ of the lowest state in the k^{th} twisted sector. Namely, we will see that when $\epsilon^{(k)}$ is positive, indicating an instability in the non-commutative gauge theory, the associated twisted sector contains tachyons, while when the sign of $\epsilon^{(k)}$ is negative the corresponding twisted sector is non-supersymmetric but does not contain tachyons. Finally, vanishing of $\epsilon^{(k)}$ (absence of pole-like IR divergences for the related gauge theory sector) is correlated with the k^{th} closed string twisted sector being supersymmetric. Indeed, this is the case iff $ka_\alpha = 0 \bmod N$ for some α, so $\epsilon^{(k)} = 0$ from (2.6).

In order to prove the previous statement, we turn now to analyze the closed string spectrum. In the k^{th} twisted sector, worldsheet bosonic complex coordinates Z^i, $Z^{\bar{i}}$ have moddings $n+\theta_i$, $n-\theta_i$, with $n \in \mathbb{Z}$ and $\theta_i = kb_i/N$; worldsheet fermionic complex fields ψ^i, $\psi^{\bar{i}}$ have moddings $n+1/2+\theta_i$, $n+1/2-\theta_i$ in the NS sector and $n+\theta_i$, $n-\theta_i$ in the R sector. The spectrum of the string, as a whole, is unchanged under the redefinition

$$\theta_i \to \theta_i + k_i \tag{2.11}$$

with $k_i \in Z$ to preserve the modding of the oscillators, and $\sum_i k_i =$ even, to guarantee that the recover the same GSO projection. One may use this freedom to put the redefined θ_i in the range $(-1,1]$, as we assume henceforth.

In R sectors, the zero point energy vanishes and the $(\text{mass})^2$ of the corresponding spacetime states is non-negative. Therefore, and due to level-matching, all states in the NS-R, R-NS and R-R sector are necessarily non-tachyonic.

In the NS-NS sector, the lightest spacetime states in the string tower are potentially tachyonic. When θ_i are in the range $(-1,1]$, the states which can become tachyonic may be described as follows. Consider say the left moving sector and define a 'groundstate' $|0\rangle$ to be annihilated by all bosonic oscillator with positive modding, and by the fermionic oscillators $\psi^i_{n+1/2+\theta_i}$ and $\psi^{\bar{i}}_{n+1/2-\theta_i}$ for $n \geq 0$

$$\psi^i_{n+1/2+\theta_i}|0\rangle = 0\,; \qquad \psi^{\bar{i}}_{n+1/2-\theta_i}|0\rangle = 0\,; \qquad \forall n \geq 0\,. \tag{2.12}$$

This coincides with the usual vacuum of the NS sector for small θ_i. When some $|\theta_i| > 1/2$ it is not the usual vacuum, since it is annihilated by some fermion oscillator with negative modding. However, it will be useful to use this state throughout the range $\theta_i \in (-1,1]$. The energy of this groundstate is given by the zero point energy contribution $E_0 = (1/2)(|\theta_1| + |\theta_2| + |\theta_3| - 1)$, and this state is odd under $(-)^F$. In order to describe the lightest states for positive and negative θ_i's in the range $(-1,1]$ in a unified way, let us define the operators

$$\begin{aligned}
\Psi^i{}_{-1/2+|\theta_i|} &= \psi^i_{-1/2+\theta^i} \qquad && \text{for} \quad \theta_i > 0 \\
\Psi^i{}_{-1/2+|\theta_i|} &= \psi^{\bar{i}}_{-1/2-\theta^i} \qquad && \text{for} \quad \theta_i < 0\,.
\end{aligned} \tag{2.13}$$

The modding of the oscillator $\Psi^i{}_{-1/2+|\theta_i|}$ is therefore $-1/2+|\theta_i|$ for any sign of θ_i. The lightest states surviving the GSO projection in the left sector are

$$\begin{array}{ll}
\Psi^1_{-1/2+|\theta_1|}|0\rangle & \alpha' m_1^2 = |\theta_2| + |\theta_3| - |\theta_1| \\
\Psi^2_{-1/2+|\theta_2|}|0\rangle & \alpha' m_2^2 = |\theta_3| + |\theta_1| - |\theta_2| \\
\Psi^3_{-1/2+|\theta_3|}|0\rangle & \alpha' m_3^2 = |\theta_1| + |\theta_2| - |\theta_3| \\
\Psi^1_{-1/2+|\theta_1|}\Psi^2_{-1/2+|\theta_2|}\Psi^3_{-1/2+|\theta_3|}|0\rangle & \alpha' m_4^2 = 2 - \sum_{i=1}^3 |\theta_i| \,.
\end{array} \tag{2.14}$$

In the range $\theta_i \in (-1, 1]$ no other state can become tachyonic. Due to level-matching, the masses of the lowest states in the NS-NS sector will be twice these ones.

The freedom (2.11) allows to set the parameters θ_i not only in the range $(-1, 1]$, but also all ≥ 0 or all < 0. Having into account that $\theta_i = kb_i/N$ and the expression of b_i in terms of a_α (2.3), we see that (2.11) is equivalent to the freedom in defining the parameters ka_α modulo N. Using these two facts, it is straightforward to show

$$\begin{array}{ll}
\alpha' m_1^2 = 2\epsilon \dfrac{ka_1}{N} + r_1 \,, & \alpha' m_2^2 = 2\epsilon \dfrac{ka_2}{N} + r_2 \,, \\
\alpha' m_3^2 = 2\epsilon \dfrac{ka_3}{N} + r_3 \,, & \alpha' m_4^2 = 2 + 2\epsilon k \dfrac{a_4}{N} - (r_1 + r_2 + r_3) \,,
\end{array} \tag{2.15}$$

with r_i certain even integers, and $\epsilon = 1$ when all $\theta_i \geq 0$ and $\epsilon = -1$ when all $\theta_i < 0$. Using (2.6) we can rewrite the eigenvalues of M as

$$\epsilon^{(k)} = -16 \prod_\alpha \sin \frac{\pi \alpha' m_\alpha^2}{2} \,. \tag{2.16}$$

We can now compare the sign of the IR pole coefficient $\epsilon^{(k)}$ and the sign of the (mass)^2 of the lightest state in the kth closed string twisted sector. From (2.14) we observe that $-1 \leq \alpha' m_\alpha^2 \leq 2$, and hence the sign of $\sin(\pi \alpha' m_\alpha^2/2)$ will coincide with that of m_α^2. If at most one of the masses m_α^2 could be negative, we would have obtained the conjectured statement: the sign of $\epsilon^{(k)}$ is positive if the associated closed string twisted sector contains a tachyon, and negative if the closed string sector is non-supersymmetric but does not contain tachyons. In order to show that at most one of the m_α^2 can be negative, it is convenient to label (without loss of generality) the parameters θ_i such that

$$|\theta_1| \leq |\theta_2| \leq |\theta_3| \,. \tag{2.17}$$

It is clear then that only m_3^2 and m_4^2 could be negative. $m_3^2 < 0$ implies that $|\theta_1| + |\theta_2| < |\theta_3|$, while $m_4^2 < 0$ implies that $|\theta_1| + |\theta_2| > 2 - |\theta_3|$. Both conditions can not be met at the same time, since they would mean that $1 < |\theta_3|$, which contradicts the fact that the parameters θ_i has been chosen in the interval $(-1, 1]$. Thus we obtain the remarkable result that the presence or not of instabilities of the non-commutative quantum field theory is correlated with the presence or not of tachyons in the closed string spectrum of the string theory realization of the configuration.

The fact that the non-commutative field theory instabilities contain some information about the closed string sector of the string theory configuration can be put to a slightly stronger test. In fact we can consider different quantum field theories associated to the same orbifold singularity, by considering D3-branes in representations of Z_N other than the regular one. The resulting field theories have a field content similar to the above with the

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only difference that the ranks of the gauge factors are arbitrary $\mathrm{U}(n_i)$,[2] and can even vanish (so the corresponding gauge factor is absent). If the instabilities of the non-commutative field theories are indeed correlated with the existence of tachyons in closed string spectrum, all field theories obtained from the same orbifold singularity (i.e. described by the same quiver diagram) should lead to the same signs for the corresponding eigenvalues.

It is easy to see that the structure of the IR poles in this more general situation is still given by (2.4), (2.5), with all the dependence on the arbitrary ranks n_i included in the normalization of $\mathrm{Tr}\, A_\mu^{(i)}$. For this it is important to realize that the gauge couplings for different gauge factors are still equal in this more general situation. Therefore, the diagonalization of (2.5) proceeds exactly as above, via linear combinations of the form (2.8), with the proviso that if some $n_i = 0$ the corresponding gauge boson disappears from the linear combination. The diagonalization hence leads to the same eigenvalues $\epsilon^{(k)}$. Hence the sign of the one-loop pole contribution is again correlated with the presence or not of tachyons in the closed string twisted sector. Note that in the basis diagonalizing the one-loop contribution, the tree level kinetic term is not diagonal. However, in the generic case of non-zero n_i's, the sign of the ϵ_k's correlated with field theory instabilities, since the latter occur at low momenta where the pole dominates the effective action. When some $n_i = 0$, the fields B_k are not all independent, and it is linear combinations of the ϵ_k that determine the sign of the pole for the independent fields, as in the example below.

To verify that our general recipe is valid even when some n_i vanish, it is interesting to further explore the extreme situation where all n_i's vanish except for one. In the non-supersymmetric case (no $a_\alpha = 0$), the quantum field theory reduces to pure $\mathrm{U}(n_j)$ Yang-Mills, coupled to adjoint complex scalars if some $b_\beta = 0$. In this situation,

$$B_\mu^{(k)} = \frac{1}{\sqrt{N}} e^{2\pi i\, jk/N} \mathrm{Tr}\, A_\mu^{(j)} \tag{2.18}$$

and our general formula (2.7) would give

$$\Delta S = \frac{g^2}{2\pi^2} \int \frac{d^4p}{(2\pi)^4} \frac{\tilde{p}^\mu \tilde{p}^\nu}{\tilde{p}^4} \frac{1}{N} \sum_{k=0}^{N-1} \epsilon^{(k)} \mathrm{Tr}\, A_\mu^{(j)}(p)\, \mathrm{Tr}\, A_\nu^{(j)}(-p)\,. \tag{2.19}$$

The coefficient should equal the contribution directly computed from the spectrum of gauge bosons and adjoint scalars. This requires a sum rule for the eigenvalues, which is happily satisfied

$$\begin{aligned} \frac{1}{N}\sum_{k=1}^{N} \epsilon^{(k)} &= \frac{1}{N}\sum_{k=1}^{N} 2\left(1 - \sum_\alpha \cos\frac{2\pi k a_\alpha}{N} + \sum_\beta \cos\frac{2\pi k b_\beta}{N}\right) \\ &= 2 + 2\sum_{\beta=1}^{3} \delta_{b_\beta,0} \end{aligned} \tag{2.20}$$

since the later is precisely the contribution to the pole from gauge bosons and adjoint scalars (if present).

[2] Cancellation of RR twisted tadpoles implies some constraints on this numbers which may be understood in terms of cancellation of cubic non-abelian anomalies in the usual commutative situation [18]. We consider such constraints to be satisfied, since they do not modify our argument.

3. String interpretation

In the previous section we analyzed both the one-loop field theory spectrum and the tree-level spectrum of the closed string theory. We found that they are related. In this section we will further explore the string origin of (2.7).

Let us consider again D3-branes in the regular representation of a type IIB $\mathbb{C}^3/\mathbb{Z}_N$ orbifold. The contribution to the effective action coming from the pole-like infra-red divergences (2.7) is not gauge invariant. However it can be completed to produce a gauge invariant expression [19, 15, 20]

$$\Delta S = \frac{1}{2\pi^2} \sum_{k=0}^{N-1} \int \frac{d^4 p}{(2\pi)^4} \frac{\epsilon^{(k)}}{\tilde{p}^4} W^{(N-k)}(p)\, W^{(k)}(-p)\,, \tag{3.1}$$

with

$$W^{(k)} = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{2\pi i\, jk/N}\, \widetilde{W}^{(j)}\,, \tag{3.2}$$

and $\widetilde{W}^{(j)}$ denoting the simplest gauge invariant open Wilson line operators [21, 22]

$$\widetilde{W}^{(j)}(p) = \mathrm{Tr} \int d^4x\; P_* \left(e^{i\, g \int_0^1 d\sigma\, \tilde{p}^\mu A_\mu^{(j)}(x+\tilde{p}\,\sigma)} \right) * e^{ipx}\,. \tag{3.3}$$

Notice that the terms $\mathrm{Tr}\,1$ in the expansion of the exponential cancel in $W^{(k)}$ for $k \neq 0$, while for $k = 0$ we have $\epsilon^{(0)} = 0$. Thus, the leading term in the expansion of (3.1) reproduces (2.7).

In the context of D-branes in B-field backgrounds, it has been shown that closed string modes couple naturally to straight open Wilson line operators [23]–[25]. The closed strings in the k^{th} twisted sector couple to gauge invariant operators formed out of fields in the adjoint of the j^{th} gauge group as [26]

$$\mathrm{Tr}(\gamma_k \lambda_j)\, \phi_k\, \mathcal{O}_j = e^{2\pi i\, jk/N} \phi_k\, \mathcal{O}_j\,, \tag{3.4}$$

where γ_k is the Chan-Paton matrix associated to the k^{th} twist and λ_j projects on the Chan-Paton indices of the j^{th} gauge group factor. Using this we observe that the operators $W^{(k)}$ defined in (3.2) will couple to closed strings in the k^{th} twisted sector. Therefore the term in the gauge theory effective action responsible for the instability (3.1) looks formally like a closed string exchange between D-branes. The closed string modes that can contribute to (3.1) must have zero spin since $W^{(k)}$ carry no spin.[3] We have seen in the previous section that tachyonic modes have always zero spin. It is interesting to notice that they are the first candidates to couple to $W^{(k)}$. Motivated by this analogy, it is natural to wonder whether $1/\tilde{p}^4$ in (3.1) can be related in some way to a closed string propagator. We will now analyze this question, generalizing the approach developed in [15] for type 0 D3-branes.

[3] More in general, the closed string modes that can couple to $W^{(k)}$ must be singlets under the unbroken subgroup of the Lorentz group.

Spin zero closed string modes will couple to a series of field theory operators weighted by appropriate α' powers. At leading order in α' they generically couple to the brane tension. In the absence of an expectation value of the B-field, the field theory operator associated to the tension of a set of D3-branes is just Tr 1. In the presence of a B-field background, Tr 1 is promoted to the simplest straight open Wilson line operator. Let ϕ be an spin zero closed string mode belonging to the k^{th} twisted sector of the orbifold $\mathbb{C}^3/\mathbb{Z}_N$. At leading α' order, ϕ will couple to the field theory operator

$$\mathcal{O}(p) = \frac{c_\phi}{(2\pi\alpha')^2} W^{(k)}(p)\,, \tag{3.5}$$

where $W^{(k)}$ is given by (3.2) and c_ϕ is a numerical factor which can be extracted from the disk amplitude with an insertion of the closed string mode ϕ and no open string insertions. At sub-leading order in α', ϕ will couple to open Wilson line operators with the insertion of non-trivial spin zero field theory operators, for example F^2. We are interested in giving a string theory interpretation to (3.1), therefore from now on we will concentrate on the coupling of ϕ to $\mathcal{O}$ as in (3.5).

In the presence of an expectation value of the B-field, open and closed string modes neither perceive the same space-time metric, nor string coupling constant [27]. The following relation holds

$$G^{-1} + \frac{\theta}{2\pi\alpha'} = \frac{1}{g + 2\pi\alpha' B}\,, \qquad G_s = g_s \left(\frac{\det G}{\det g}\right)^{1/4}\,, \tag{3.6}$$

where g and g_s (G and G_s) denote the closed (open) string metric and coupling constant respectively. The coupling of ϕ to the field theory operator $\mathcal{O}$ is described by the action

$$S = \frac{k_D}{2\pi G_s} \int \frac{d^4p}{(2\pi)^4} \sqrt{\det G}\, \phi(-p)\, \mathcal{O}(p)\,, \tag{3.7}$$

where D is the number of dimensions where the twisted field ϕ can propagate. In the generic $\mathbb{C}^3/\mathbb{Z}_N$ orbifold, we have $D = 4$. However, depending on the particular orbifold model and twisted sector we can also have $D = 6, 8, 10$. k_D is the gravitational coupling constant in D-dimensions

$$k_D^2 \sim g_s^2 \alpha'^{(D-2)/2}\,. \tag{3.8}$$

The contribution to the field theory effective action from the exchange of ϕ between two D3-branes separated by a distance r will be

$$\Delta S = \frac{k_D^2}{(2\pi G_s)^2} \int \frac{d^4p}{(2\pi)^4} \det G\, \mathcal{O}^\dagger(p)\, \mathcal{O}(-p) \left(\frac{1}{\sqrt{\det g}} \int \frac{d^d p_\perp}{(2\pi)^d} \frac{e^{ip_\perp r}}{p_\perp^2 + M^2}\right), \tag{3.9}$$

where p and $p_\perp$ denote the components of the momentum along and transversal to the D3-brane respectively. The expression in brackets is the closed string propagator in $d = D - 4$ dimensions. The above expression (3.9) is valid for NS-NS fields. R-R exchange results with an effective action with an opposite sign. M^2 acts as an effective mass term for the propagation in the transverse dimensions

$$M^2 = m^2 - g^{\mu\nu} p_\mu p_\nu = m^2 - p^2 + \frac{\tilde{p}^2}{(2\pi\alpha')^2}\,, \tag{3.10}$$

with m^2 being the mass of ϕ in D-dimensions. We have used $g^{-1} = G^{-1} - \theta G\theta/(2\pi\alpha')^2$, and denoted $p^2 = p\cdot G^{-1}\cdot p$ and $\tilde{p}^2 = -\tilde{p}\cdot G\cdot\tilde{p}$. The non-commutative field theory limit consists in sending $\alpha' \to 0$ while keeping G, θ and $g^2_{YM} = 2\pi G_s$ fixed. It is in this limit where we want to analyze (3.9). Notice that when $B = 0$, the leading α' effect is related to $m^2 \sim 1/\alpha'$. However when the field theory limit is taken at fixed non-zero θ, m^2 becomes a sub-leading effect if the closed string mode carry momentum along the non-commutative directions [28, 24]. In the following we will neglect the dependence of M^2 on p^2, suppressed by two powers of α' with respect to $\tilde{p}^2$, but will retain its dependence on m^2.

It is convenient to introduce $y = 2\pi\alpha' p_\perp$, $u = r/(2\pi\alpha')$ and set $G = \eta$, the Minkowski metric. Using (3.6) and (3.5) we obtain[4]

$$\Delta S = \int \frac{d^4p}{(2\pi)^4}\, W^{(N-k)}(p)W^{(k)}(-p)\, f(\tilde{p}, u) \tag{3.11}$$

with

$$f(\tilde{p}, u) = \frac{|c_\phi|^2\, k_D^2}{(2\pi g_s)^2(2\pi\alpha')^{D-2}} \int \frac{d^dy}{(2\pi)^d} \frac{e^{iyu}}{y^2 + \tilde{p}^2 + (2\pi\alpha' m)^2}\,. \tag{3.12}$$

The fraction in front of the integral is independent of g_s and proportional to $\alpha'^{1-D/2}$. The integral is finite when $\alpha' \to 0$. Thus $f(\tilde{p}, u)$ diverges when we send $\alpha' \to 0$. We could however define a finite contribution by expanding the integral in powers of $2\pi\alpha' m$ to order $\alpha'^{D/2-1}$

$$f(\tilde{p}, u)|_{\mathcal{O}(\alpha'^0)} \sim |c_\phi|^2\, (\alpha' m^2)^{(D/2)-1} \int \frac{d^dy}{(2\pi)^d} \frac{e^{iyu}}{(y^2 + \tilde{p}^2)^{D/2}}\,. \tag{3.13}$$

With this prescription (3.11) defines a finite contribution to the field theory 1-loop effective action. The parameter u, measuring the separation between the two D3-branes, has the field theory interpretation of a mass for the degrees of freedom circulating in the loop. In the non-planar field theory loops the scale $1/\tilde{p}$ acts as ultraviolet cutoff. Thus when $\tilde{p}u > 1$ the contribution of non-planar graphs is strongly suppressed. We observe that this pattern is reproduced by (3.13). The parameter u was introduced in order to have a well-defined closed string propagator. Now that we have derived from $f(\tilde{p}, u)$ an expression candidate to have a field theory interpretation, we can let $u \to 0$. The result is

$$f(\tilde{p}, 0)|_{\mathcal{O}(\alpha'^0)} \sim \frac{|c_\phi|^2\, (\alpha' m^2)^{(D/2)-1}}{\tilde{p}^4}\,. \tag{3.14}$$

Hence the term $1/\tilde{p}^4$, source of the pole like IR divergent behavior, can be traced back to a closed string propagator independently of the dimension D in which the closed string modes live. This does not mean that the decoupling limit fails in the non-commutative case, since the IR singularities do not have kinetic part. Therefore they do not force the introduction of new degrees of freedom.

We have seen that considering the exchange of a single closed string mode between D-branes, we are able to reproduce the field theory effective action (3.1). However this is achieved *up to an overall factor*. It is crucial to notice that all closed string modes (of

[4] See [29] for an alternative derivation of an analogous expression in the context of bosonic string theory.

spin zero or spin zero combinations of them) contribute as in (3.11) and (3.14). Indeed by open/closed channel duality the contribution from the lowest modes in the open string tower are mapped to the contribution from an infinite number of closed string oscillators. Thus while the functional dependence on $\tilde{p}$ can be related to a single closed string propagator, the numerical factor $\epsilon^{(k)}$ is a collective effect of the whole closed string tower.

The surprising fact is that, although it includes the contribution from all closed string modes, $\epsilon^{(k)}$ is at the same time determined by the lowest states in the tower. These two facts can be reconciled in the following heuristic way. Modes in the NS-NS sector contribute with different sign to those in the R-R sector to the exchange between D-branes. When the theory is supersymmetric both towers must cancel since we know that in this case $\epsilon^{(k)} = 0$. This is actually easy to see also from the string approach: The Wilson line operators $W^{(k)}$ generalize the coupling of the closed string modes to the brane tension in the case of non-zero B-field; Thus, when the theory is supersymmetric the condition of no force between the branes implies that $\epsilon^{(k)} = 0$ (see below). In particular, the untwisted sector of the orbifold models is supersymmetric and hence $\epsilon^{(0)} = 0$. This suggests that, in the absence of supersymmetry, $\epsilon^{(k)}$ can be considered a measurement of the misalignment between the NS-NS and R-R towers. On the other hand, the expression of $\epsilon^{(k)}$ in terms of the masses of the lowest states in the NS-NS tower (2.16) can be also seen as a measure of the misalignment between both towers, since the lightest states in the R-R sector are massless. At any rate, we are uncovering a new relation fulfilled by the closed string spectrum, reminiscent of some results based in modular invariance (in that they related properties of the whole string tower with properties of the lightest modes), which would be very interesting to re-derive from first principles.

The gauge invariant effective action (3.1) is also valid in the case we are considering D3-branes in a generic the representation of the orbifold group. The only difference then is that the Wilson line operators $\widetilde{W}^{(j)}$ should be redefined as

$$\widetilde{W}^{(j)}(p) = \mathrm{Tr} \int d^4x \; P_* \left(e^{i\, g \int_0^1 d\sigma\, \tilde{p}^\mu A_\mu^{(j)}(x+\tilde{p}\,\sigma)} - 1 \right) * e^{ipx} \,. \tag{3.15}$$

Contrary to the case of regular D3-branes, the components $\mathrm{Tr}_j 1 = n_j$ of $\widetilde{W}^{(j)}$ do not cancel in the definition (3.2). Therefore, in order that (3.1) does not contain spurious terms, they have to be explicitly subtracted.

Notice that the closed string modes couple to the complete Wilson line operators. This does not affect our string derivation of the field theory effective action because it was only valid at non-zero momentum in the non-commutative directions. Subtracting or not the constant term n_j from $\widetilde{W}^{(j)}$ affects only its zero momentum component.

However, the string derivation provides an interpretation for the terms proportional to n_j. As we have seen these terms correspond to the coupling of the closed string modes to the brane tension. Therefore channel duality implies that by considering the exchange of all closed string modes, in the $\alpha' \to 0$ limit, we obtain the contribution of one-loop vacuum diagrams to the field theory effective action

$$\Gamma = -\frac{1}{2}\, V\, \mathrm{Tr}\, (-1)^F \int \frac{d^4p}{(2\pi)^4} \log p^2 \,. \tag{3.16}$$

where V represents the infinite 4-dimensional volume. We will now show explicitly that (3.16) coincides with (3.1) when the Wilson line operators are replaced by their constant component. Substituting $\widetilde{W}^{(j)} \to (2\pi)^4 n_j \delta^{(4)}(p)$ in (3.1) we obtain

$$\frac{1}{2\pi^2}\Lambda^4 \left(\sum_{k=0}^{N-1} \epsilon^{(k)} v^{(N-k)} v^{(k)} \right) \int \frac{d^4p}{(2\pi)^4} \left((2\pi)^4 \delta^{(4)}(p) \right)^2 , \tag{3.17}$$

where we have defined $v^{(k)} = (1/\sqrt{N}) \sum_{j=1}^{N} e^{2\pi i k j/N} n_j$. The delta function that substitute the Wilson line operators set p and thus $\tilde{p}$ to zero. Therefore the factor $1/\tilde{p}^4$ in (3.1) will give rise to a divergence. Since we want to compare (3.17) with the planar contribution coming from vacuum loops, we have replaced in it $1/\tilde{p}^4|_{\tilde{p}=0}$ by Λ^4. We interpret Λ as an UV regulator.

The integral in (3.16) is quartically divergent. It can be evaluated by introducing an UV cutoff

$$-\frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \log p^2 \to \frac{1}{8\pi^2}\int_0^\infty \frac{dt}{t^3} e^{-1/(4\Lambda^2 t)} = \frac{1}{2\pi^2}\Lambda^4 . \tag{3.18}$$

The cutoff has been chosen by analogy to how $1/\tilde{p}$ regulates the UV region of non-planar integrals. This is necessary in order to make the substitution $1/\tilde{p} \to \Lambda$ in (3.17) meaningful. We also have

$$\sum_{k=0}^{N-1} \epsilon^{(k)} v^{(N-k)} v^{(k)} = \sum_{i,j=1}^{N} M_{ij} n_i n_j = \mathrm{Tr}(-1)^F , \tag{3.19}$$

where the matrix M_{ij} is given in (2.5). Finally the integral in (3.17) is just $\int d^4x = V$ written in momentum space. This shows that (3.17) exactly reproduces the contribution from the field theory 1-loop vacuum diagrams, and how this depend on the coefficients $\epsilon^{(k)}$. In particular $\epsilon^{(k)}$ govern in a similar way the term in the effective action responsible for the instabilities (3.1) and the contribution from the vacuum loops.

We would like to end this section with a comment on the cases with scalars in the adjoint representation. Adjoint scalars, as gauge bosons, get pole-like IR corrections to their self-energy. This can be incorporated in the string derivation by just generalizing the definition of the Wilson line operators to

$$\widetilde{W}^{(j)}(p) = \mathrm{Tr}\int d^4x \; P_* \left(e^{i\, g \int_0^1 d\sigma \left(\tilde{p}^\mu A_\mu^{(j)}(x+\tilde{p}\,\sigma) + y_i \phi_i^{(j)}(x+\tilde{p}\sigma) \right)} \right) * e^{ipx} , \tag{3.20}$$

where y_i label as before the momentum in the directions transverse to the D3-branes and ϕ_i denote the adjoint (real) scalars. Indeed, it has been shown [24, 25] that (3.20) are the field theory operators to which closed string modes naturally couple when adjoint scalars are present. For a detailed derivation of how (3.20) gives rise to the pole-like IR corrections to the self-energy of adjoint scalars on type 0 D3-branes see [15]. The discussion there generalizes straightforwardly to the more generic orbifold models considered in this paper.

4. Discussion

String theories without space-time supersymmetry tend to have tachyons in the spectrum. This statement can be done more precise by analyzing the torus partition function

$$Z = \int \frac{d^2\tau}{\tau_2} \mathrm{Tr}(-)^F q^{L_0} \bar{q}^{\bar{L}_0} \,, \tag{4.1}$$

where $q = e^{2\pi i\tau}$ and $\tau = \tau_1 + i\tau_2$ is the complex modulus of the torus. This integral can have both UV ($\tau_2 \to 0$) and IR ($\tau_2 \to \infty$) divergences. It is easy to see that an IR divergence can only appear if there are tachyons in the spectrum. The trace in (4.1) reduces in the UV to the supertrace over the number of degrees of freedom. Modular invariance of (4.1) relates the UV and IR regimes. As a result, the absence of closed string tachyons requires that the density of space-time bosons and fermions cancel to a great accuracy, although both quantities will generically grow exponentially with the energy [30]. String theories without this sort of "asymptotic supersymmetry" will have tachyons in the spectrum. This provides a very precise relation between tachyons and absence of supersymmetry in string theory.

String theory regularizes the UV region of the torus partition function by restricting the integration to the fundamental domain of the modular group. But possible IR divergences remain. String theory translates the possible divergence on vacuum diagrams because of the growing density of states into an IR divergence. This is a particular case of the general situation: all divergences in string theory have an IR interpretation. Therefore, contrary to UV divergences which are susceptible of being regularized, they have a physical meaning and can not be discarded.

The extremely remarkable thing is that a version of all these features seems to survive in non-commutative field theories, without the inclusion of gravity. First, non-planar graphs of non-commutative field theories translate UV into IR behavior. In particular, they transform potential UV divergences into IR divergences. This phenomenon however does not take place in the planar sector of the theory. As an interesting example, we have seen that the quartic divergences of the field theory vacuum energy do have an infrared translation in (3.1). Second, absence of supersymmetry implies strong modifications of the spectrum of the theory at low momentum. This is specially neat for non-commutative gauge theories. Indeed, the effective action (3.1) implies the presence of unstable modes in one to one correspondence with closed string tachyons for those gauge theories which can be derived as limits of string theory. Although a deeper understanding is lacking of why these phenomena are present in non-commutative field theories, it agrees with the proposal that non-commutative geometry is able to encode in an effective way crucial aspects of quantum gravity [31].

We would like to point out that the correlation between the existence (or not) of instabilities in the non-commutative field theory and the existence (or not) of tachyons in the closed sector of the underlying string theory, seems to be quite general, and not restricted to the family of non-supersymmetric orbifold theories we have been considering. In fact, it is possible to find such a relation in particular examples of non-commutative field theories arising on the volume of stable non-BPS branes in supersymmetric theories. In particular, one may test the issue by considering the configurations studied in [32], namely compactification of type-IIB theory on the orbifold limit $T^4/\mathbb{Z}_2$ of K3, with a stack of n (stable at short radii) non-BPS D4-branes wrapped on a 1-cycle passing through fixed points. The field theory on the 4d non-compact directions of the D-branes has gauge group $\mathrm{U}(n)$, four Majorana fermions in the adjoint and one adjoint complex scalar. Turning on a NSNS 2-form field in the non-compact directions makes this theory non-commutative; the field content reveals that there is a one-loop correction to the mass of gauge bosons, but no instability. This agrees with our claim, since the closed string sector does not contain tachyons.

We should stress that although the closed string sector is supersymmetric, the one-loop correction does not vanish, as was the case for D3-branes at orbifold singularities. We blame this on the non-BPS character of the corresponding D-branes, while for D3-branes at orbifold singularities, the D3-branes were in a sense, still BPS states of the theory before orbifolding.

$\mathcal{N}=4$ non-commutative $\mathrm{U}(n)$ theory at finite temperature was analyzed in [12]. Temperature breaks supersymmetry but at the same time acts as an UV regulator. As a consequence this theory presents a regularized version of UV/IR mixing. The strength of UV/IR mixing effects grows with the temperature, such that for T above a critical temperature $T_c \sim 1/\sqrt{g\theta}$ collective excitations of tachyonic nature appear in the system. An important characteristic of this tachyonic branch is that $E^2(p) \to 0$ for low momenta. Namely, contrary to the behavior in purely non-supersymmetric theories, UV/IR effects give rise to instabilities without the appearance of IR divergences. Let us consider the string embedding of this theory, namely, a D3-brane in Type IIB at finite temperature. It is well known that string theory becomes unstable at the Haggedorn temperature, $T_H \sim 1/\sqrt{\alpha'}$. Following the spirit of this paper it is tempting to try to relate both instabilities, and in particular both critical temperatures. We find however a problem. In the $\alpha' \to 0$ limit we have $T_H \to \infty > T_c$. This suggests that the relation between non-commutative instabilities and closed string tachyons could make sense only for field theory instabilities where $E^2 < 0$ as $p \to 0$. Indeed, when the field theory temperature is sent to infinity, an IR divergence is generated [12].

It is also interesting to ask to what extent the non-commutative nature of the field theory is relevant for the connection with closed string tachyons. Non-commutative gauge theories exists only for unitary gauge groups and matter in the fundamental, bifundamental and adjoint representations. We have seen in what remarkable way bifundamental and adjoint degrees of freedom can be related with closed string tachyons for theories on D3-branes. Fundamental matter, if present, does not contribute to this relation since it does not give rise to non-planar diagrams. We could ask then whether this relation can be extended to gauge groups and matter content which do not admit a non-commutative version.[5] In fact, one may consider quantities like the number of bosonic minus fermionic degrees of freedom, weighted in a suitable way to account for gauge quantum numbers (say,

[5]Using a generalization of the Seiberg-Witten map, it has been proposed a way of defining non-commutative theories with arbitrary gauge group and matter content [33]. However a string realization of this setup does not exist.

matrices like (2.5)) in any field theory on D3-branes, and try to correlate the corresponding signs with closed string tachyons. Interesting examples to analyze would be cases including orientifold projections. We leave this as an open question for further study.

Let us come back to the particular setup of orbifold singularities, and discuss the relation between our statements and other approaches in the literature. The fate of twisted closed string tachyons of $\mathbb{C}/\mathbb{Z}_N$ and $\mathbb{C}^2/\mathbb{Z}_N$ (N odd) non-supersymmetric orbifolds has been studied [1]. It was argued that the end point of the condensation will generically be flat space. Information about this process can be obtained by placing D-branes probes at the orbifold point. The Higgs branch of the probe gauge theory, associated to expectation values of the bifundamental fields, reproduces the geometry of the orbifold. As the tachyon starts condensing new tree level couplings appear in the gauge theory. Among them are masses for the bifundamental fields which, due to coming from twisted sectors, must add up to zero (see (3.4)). Thus at least one bifundamental must become unstable. As a result the Higgs branch is modified and represents a geometry in which the tip of the cone has been smoothed out. All the analysis is done at the classical level, keeping α' finite.

Contrary to that, we consider non-commutative D3-branes at the tip of an $\mathbb{C}^3/\mathbb{Z}_N$ orbifold in the $\alpha' \to 0$ limit. In this situation the branes decouple from the gravity sector. The gauge theory instability is a 1-loop effect. In addition, the instability affects the adjoint fields and not the bi-fundamentals. Type 0 D3-branes [34, 35] and D3-branes at the tip of non-supersymmetric orbifold singularities [36] has been also studied in the decoupling limit for the case of zero B-field. For models with a Coulomb branch, a Coleman-Weinberg effective potential for the diagonal components of the scalar fields was calculated. This potential, associated with twisted operators under the orbifold action, showed an instability towards the separation of the branes. The instability could be considered as inherited from the original twisted string tachyons. However, the correspondence in this case was far from one to one. First it required configurations with several branes. Then it was only manifest when there was a Coulomb branch in the theory, representing dimensions along which the branes could separate. Non-supersymmetric $\mathbb{C}^3/\mathbb{Z}_N$ models without adjoint scalars do not exhibit these instabilities.

One could hope that the D3-brane models in the decoupling limit could serve as toy models of the parent string theory instabilities. In the case with zero B-field, as we have mentioned, the field theory dynamics is not rich enough to fulfill this expectation. The case in which the decoupling of gravity limit is taken at non-zero B is potentially different. The B-field allows the field theory to keep some stringy properties. It is natural then to wonder if the gauge theory instabilities will have a related fate to that of closed string tachyons. Notice that not only there is a one to one correspondence between both instabilities, but also both have a related origin in the appearance of divergences due to the absence of supersymmetry. We will not attempt to answer this question here. Understanding of the dynamics of the non-commutative instability is beyond the scope of this paper. However we would like to add some comments.

A non-commutative theory can be alternatively formulated in terms of a matrix theory of infinite-dimensional matrices in an space-time with two dimensions less. In string language this means that a Dp-brane in a B-field background possesses a non-zero den-

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sity of D$(p-2)$-branes dissolved on it. The non-commutative instability translates in this language into an instability of the smeared D$(p-2)$-brane distribution [19, 12]. The origin of the instability is that the forces between these smaller branes do not cancel in the non-supersymmetric case, which suggest that there is no stable vacuum [19]. This situation relates with the cases treated in this paper where there are fractional branes present. Namely branes fixed at the orbifold point, which do not have an analog in the flat supersymmetric theory. For D3-branes at orbifold singularities, the Chern-Simons coupling between the R-R two-forms $C_k^{(2)}$, for $k = 0, \ldots, N-1$, and the B-field is proportional to [26]

$$\mathrm{Tr}\,\gamma_k \int_4 C_k^{(2)} \wedge B = \left(\sum_j n_j e^{2\pi i j k/N}\right) B \int_2 C_k^{(2)} \tag{4.2}$$

where we have used (3.4) and the fact that B belongs to the untwisted sector. We observe that when there are fractional branes present, i.e. the ranks of the N gauge group factors do not all coincide, the B-field acts as a source for some of the twisted R-R 2-forms. Hence the initial brane configurations contains a background charge which under the effective action (3.1) tends to collapse, giving likely rise to no stable vacuum [19].

A potentially different case is that of branes in the regular representation of the orbifold group. In this case $n_j = n$ for all j in (4.2) and the B-field acts as a source for only the untwisted R-R 2-form. As we have seen in (2.16), the associated field theory sector behaves as supersymmetric and hence stable. This suggest that perhaps in this case the field theory has an stable vacuum. It is natural then to make the following conjecture: whenever the bulk tachyons condense and we end up with type-IIB string theory on flat background the 'twisted' sector tachyons of the field theory will condense as well and we will end generically with U(n) $\mathcal{N} = 4$ SYM. In that case the role of field theory the instability would be to avoid that the regular brane charge could decompose in its constituent physical branes. It could be extremely interesting and useful if, by using such a conjecture and by proving that the field-theory tachyons condense as proposed, we could improve our understanding of closed string tachyon condensation.

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